INTERNATIONAL JOURNAL OF ADVANCED STUDIES IN COMPUTER SCIENCE AND ENGINEERING
IJASCSE VOLUME 6 ISSUE 2, 2017
02/28/2017

Modeling, Riccati-Sylvester Decoupling and Digital Multiloop Control of 3-DoF Gimbaled Stabilizing Platform (Part-II)

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***Abstract*—This article addresses the design and control of a 3 Degrees of Freedom (DoF) gimbaled stabilizing platform. Order reduction is performed using Riccati-Sylvester transform for the linear model of the plant, which is Jacobean linearized version of the nonlinear dynamics derived in part I of this work. An algorithm is presented for the transformation and is applied to the system. The decoupled system is simulated and a novel adaptive control algorithm is designed to regulate the position of the platform in 3-DoF. The experimental validation of the theoretically proposed controller is also presented by implementing the discrete time realization of the control algorithm using digital controller interfaced in the real-time, using MATLAB/Simulink, in the Rapid Control Prototyping (RCP) mode of operation. The feasibility of the proposed design is theoretically and experimentally verified by its efficiency comparison with the classical techniques and satisfactorily stable closed loop responses.**

Keywords-Riccati-Sylvester transformation; 3-DoF-gimbal; stabilizing plateform; Rapid Control Prototyping; adaptive control

I. INTRODUCTION

Inertial stabilized platform is one of the most important parts of modern tracking system. The areas of applications include but are not limited to defense, aerial photography, satellite imaging, industrial measurements etc. In platform stabilization system, the platform, where any desired object can be placed, is to be maintained at a fixed reference level although there is change in system dynamics and position [1]. A 3-Axis Gimbal structure is used to inertially stabilize a platform which can also be used to track a fixed or moving point in space with the help of other sensors [2].

A lot of research work has been done owing to the importance of this system in the field and its challenging dynamic behavior to be controlled. However, the detailed modeling of this system, including the intricate geometric relations for the moment of inertia of gimbals, has not emphasized much in the literature. Secondly, the dynamics of the system are not only nonlinear but also coupled. Riccati-Sylvester differential equations have already been a topic of interest for the disturbance decoupling and optimal decentralized control problems [3]. They have also been successfully employed in the problems involving observer design [4,5], suboptimal tracking control [6] and nonlinear systems with mismatched uncertainties [7]. The real domain of application of Riccati-Sylvester differential equations is the Riccati-Sylvester transform that can be used to partially, fully or selectively decoupled a given dynamic system [8]. This work considers a procedure of Riccati-Sylvester transformation for selective decoupling of the system dynamics.

In the part II of this research, algorithm for Riccati-Sylvester transformation is presented followed by its application to 3-DoF gimbal stabilizing platform.

II. EXPERIMENTAL SETUP AND SYSTEM DYNAMICS

A 3-DoF stabilizing gimbal platform is shown in Fig. 1. This physical setup consists of four parts: a fixed frame, an outer gimbal, an inner gimbal and a stabilizing platform. The linearized system quadruple, as derived in Part I of this work, is given by (1).

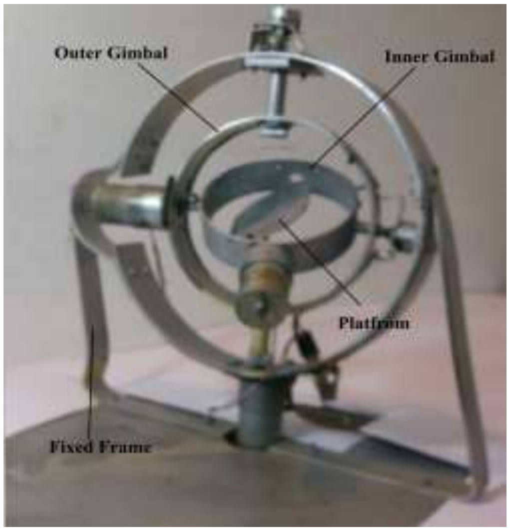

Figure 1. 3-DoF gimbal platform hardware setup.

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.0667 & 0 & 0 & 0 & 0 & 533.34 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1157 & 0 & 0 & 0 & 28.9157 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0823 & 0 & 0 & 13.7159 \\ 0 & -35.7143 & 0 & 0 & 0 & 0 & -571.4286 & 0 & 0 \\ 0 & 0 & 0 & -35.7143 & 0 & 0 & 0 & -571.4286 & 0 \\ 0 & 0 & 0 & 0 & 0 & -35.7143 & 0 & 0 & -571.4286 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 71.4286 & 0 & 0 \\ 0 & 71.4286 & 0 \\ 0 & 0 & 71.4286 \end{bmatrix}, C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, D=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1)$$

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III. RICCATI-SYLVESTER TRANSFORM

The linear system dynamics (1) are not only coupled but also of the order 9. It makes the problem very difficult to be handled. However, Riccati-Sylvester (RS) transformation only can help us to decouple the dynamics but also to reduce the order of the system dynamics [8]. Let's consider a general state space system of order n.

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x}+B\underline{u} \\ \underline{y} &= C\underline{x}+D\underline{u} \end{aligned} \quad (2)$$

According to the RS transformation method, $\underline{z}^{(i)}$ is a new state vector, similar to $\underline{x}$ with first i states completely decoupled from each other and the rest of the system being coupled subsystem if

$$\underline{z}^{(i)} = K_i \underline{z}^{(i-1)} \tag{3}$$

Where: $\underline{z}^{(0)} = \underline{x}$ =given state vector, $i = 1, 2..n$ and in recursive algorithm (3) for $i = 1$,

$$K_i = R_i \tag{4}$$

$$R_i = \begin{bmatrix} 1 & L_i \\ -P_i & -P_i L_i + I_{im} \end{bmatrix}_{n\times n} \tag{5}$$

And for $i > 1$,

$$K_i = \begin{bmatrix} I_i & \underline{0} \\ \underline{0} & R_i \end{bmatrix}_{n\times n} \tag{6}$$

$$R_i = \begin{bmatrix} 1 & L_i \\ -P_i & -P_i L_i + I_{im} \end{bmatrix}_{(n-i)\times(n-i)} \tag{7}$$

Consider a matrix $A_c^{(i)}$ given by,

$$A_c^{(i)} = \begin{bmatrix} A_{c11}^{(i)} & A_{c12}^{(i)} \\ A_{c21}^{(i)} & A_{c22}^{(i)} \end{bmatrix} \tag{8}$$

Order of square matric $A_{c11}^{(i)}$ is equal to the number of state being decoupled in the i^{th} iteration, which is 1 in our case. For $i = 1$,

$$A_c^{(1)} = A \tag{9}$$

For $i > 1$,

$$A^{(i-1)} = \begin{bmatrix} \Delta_{(i-1)\times(i-1)} & 0 \\ 0 & A_c^{(i)} \end{bmatrix} \tag{10}$$

In (10), Δ = subsystem matrix that has been decoupled in previous iteration or is intended not to be decoupled. During each of the i^{th} iteration, we get a new system matric $A^{(i)}$ with the first i states decoupled. This new matrix is given by,

$$A^{(i)} = K_i A^{(i-1)} K_i^{-1} \tag{11}$$

Here $A^{(0)} = A =$ the given system matrix. In (5), im is the order of square matrix $A_{c22}^{(i)}$ in (6). P_i is the solution of matrix Differential Riccati Equation (DRE),

$$\dot{P}_i = -P_i A_{c11}^{(i)} - P_i A_{c12}^{(i)} P_i + A_{c21}^{(i)} + A_{c22}^{(i)} P_i \tag{12}$$

And L_i is the solution of matrix Differential Sylvester Equation (DSE),

$$\dot{L}_i = -(A_{c11}^{(i)} + A_{c12}^{(i)} P_i) L_i + A_{c12}^{(i)} + L_i(-P_i A_{c12}^{(i)} + A_{r22}^{(i)}) \tag{13}$$

If we want to decouple each of the first d states simultaneously, then a composite Riccati-Sylvester transformation matrix K_d can be obtained as,

$$K_d = \prod_{i=1}^{d} K_i \tag{14}$$

And inverse of K_d in (14) is given by,

$$K_d^{-1} = \prod_{i=1}^{d} K_i^{-1} \tag{15}$$

The new decoupled state space system is given by:

$$\begin{aligned} \dot{z}^{(d)} &= K_d A K_d^{-1} z^{(d)} + K_d B u \\ y &= C K_d^{-1} z^{(d)} + D u \end{aligned} \tag{16}$$

A. Decoupling Dynamics of linearized 3-DoF gimbal platform

The RS algorithm described in (2) through (16) will now be applied to 3-DoF gimbal system quadruple (1). Let us partition the system matrix in (1) as in (17). This partitioning is based on the fact that A_4 is already diagonal and decoupled, hence it is partitioned out. Now if the first six states are to be decoupled from each other and from the rest of the system then A_1 would become diagonal and $A_2 = A_3 = \underline{0}$. Hence, we can achieve a complete decoupling by decoupling first six states, recursively in turn. So, we have to use six iterations of RS transform for $i = 1$ to 6 . We perform the partitioning (6) and get (18).

$$A=\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}=\left[\begin{array}{cccccc|ccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.0667 & 0 & 0 & 0 & 0 & 533.340 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1157 & 0 & 0 & 0 & 28.9157 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0823 & 0 & 0 & 13.7159 \\ \hline 0 & -35.7143 & 0 & 0 & 0 & 0 & -571.42 & 0 & 0 \\ 0 & 0 & 0 & -35.714 & 0 & 0 & 0 & -571.42 & 0 \\ 0 & 0 & 0 & 0 & 0 & -35.714 & 0 & 0 & -571.42 \end{array}\right] \quad (17)$$

$$A_c^{(1)}=A,\ A_{c11}^{(1)}=0,\ A_{c12}^{(1)}=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T,\ A_{c21}^{(1)}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},\ A_{c22}^{(1)}=\begin{bmatrix} -1.066 & 0 & 0 & 0 & 0 & 533.3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.115 & 0 & 0 & 0 & 0 & 28.91 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.082 & 0 & 0 & 13.71 \\ -35.71 & 0 & 0 & 0 & 0 & -571.4 & 0 & 0 \\ 0 & 0 & -35.71 & 0 & 0 & 0 & -571.4 & 0 \\ 0 & 0 & 0 & 0 & -35.71 & 0 & 0 & -571.4 \end{bmatrix} \quad (18)$$

1) Decoupling of the first state, i=1

Using (12) we get the differential Riccati equation, which is solved using the numerical technique and we get the solution given by,

$$P_1=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (19)$$

The solution curves generated by Matlab for the DRE are shown in Fig. 2.

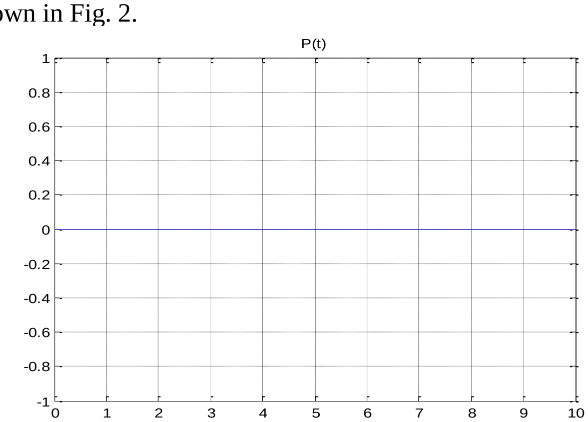

Figure 2. The solution curves for DRE, i=1.

Using (13) we get the differential Sylvester equation, which is solved using the numerical technique and we get the solution given by,

$$L_1=\begin{bmatrix} 0.0291 & 0 & 0 & 0 & 0 & 0.0271 & 0 & 0 \end{bmatrix} \quad (20)$$

The solution curves generated by Matlab for the SRE are shown in Fig. 3. Now the matrix K_1 can be obtained using (4) and (5). It is given by (21). Using (11) we get the first state decoupled system matrix given by (22)

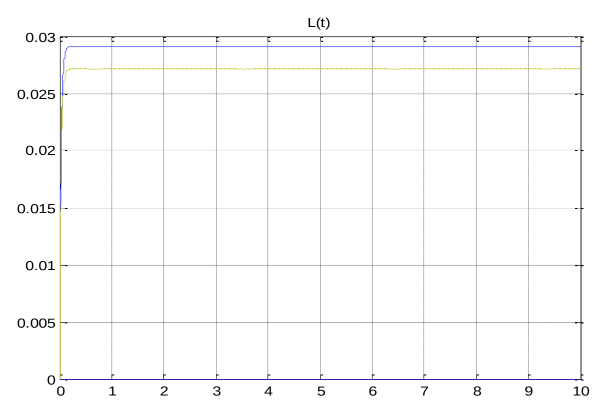

Figure 3. The solution curves for DSE, i=1.

2) Decoupling of the second state, i=2

Using (12) we get the differential Riccati equation, which is solved using the numerical technique and we get the solution given by,

$$P_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.0668 & 0 & 0 \end{bmatrix}^T \tag{23}$$

Using (13) we get the differential Sylvester equation, which is solved using the numerical technique and we get the solution given by,

$$L_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1.0685 & 0 & 0 \end{bmatrix} \tag{24}$$

$$K_1 = R_1 = \begin{bmatrix} 1 & 0.0291 & 0 & 0 & 0 & 0 & 0.0271 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

$$A^{(1)} = \begin{bmatrix} 0 & -0.0004 & 0 & 0 & 0 & 0 & -0.0054 & 0 & 0 \\ 0 & -1.0667 & 0 & 0 & 0 & 0 & 533.34 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1157 & 0 & 0 & 28.916 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0823 & 0 & 0 & 13.715 \\ 0 & -35.714 & 0 & 0 & 0 & 0 & -571.42 & 0 & 0 \\ 0 & 0 & 0 & -35.714 & 0 & 0 & 0 & -571.42 & 0 \\ 0 & 0 & 0 & 0 & 0 & -35.714 & 0 & 0 & -571.42 \end{bmatrix} \tag{22}$$

The solution curves generated by Matlab for the DRE and shown in Fig. 4

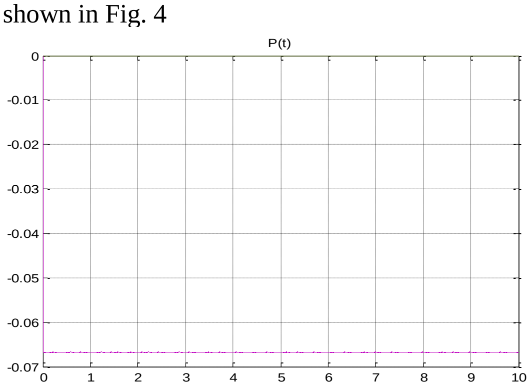

Figure 4. The solution curves for DRE, i=2.

The solution curves generated by Matlab for the DSE and shown in Fig. 5.

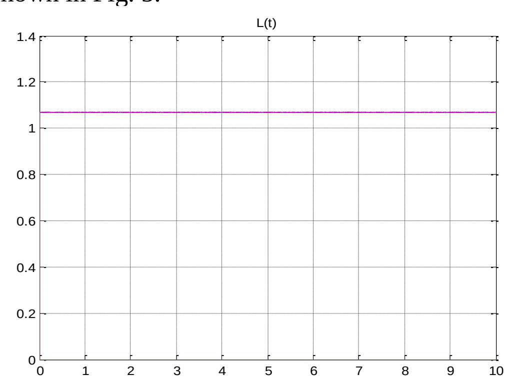

Figure 5. The solution curves for DSE, i=2.

Now the matrices R_2 and K_2 can be obtained using (6) and (7). These are given by (25).

$$R_2 = \begin{bmatrix} 1.0714 & 0 & 0 & 0 & 0 & 1.0685 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0.0668 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}, K_2 = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0714 & 0 & 0 & 0 & 0 & 1.0685 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0.0668 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad (25)$$

3) Decoupling of the states for i=3 to 6

A procedure similar to the one above can be performed for rest of the states and we get the final composite decoupling matrix (26). The decoupled system matrix is given by (27). The corresponding input and output matrices are given by (28).

$$K_d = \prod_{i=1}^{6} K_i = \begin{bmatrix} 1 & 0.0291 & 0 & 0 & 0 & 0 & 0.0271 & 0 & 0 \\ 0 & 1.0714 & 0 & 0 & 0 & 0 & 1.0685 & 0 & 0 \\ 0 & 0 & 1 & 0.5201 & 0 & 0 & 0 & 0.0263 & 0 \\ 0 & 0 & 0 & 1.0032 & 0 & 0 & 0 & 0.0509 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1.0643 & 0 & 0 & 0.0256 \\ 0 & 0 & 0 & 0 & 0 & 1.0015 & 0 & 0 & 0.0241 \\ 0 & 0.0668 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.0627 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0626 & 0 & 0 & 1 \end{bmatrix} \quad (26)$$

$$\tilde{A} = K_d A K_d^{-1} = \left[\begin{array}{cccccc|ccc} \boxed{0} & 0 & 0 & 0 & 0 & 0 & 0.005 & 0 & 0 \\ 0 & \boxed{-36.68} & 0 & 0 & 0 & 0 & 0.030 & 0 & 0 \\ 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & -0.004 & 0 \\ 0 & 0 & 0 & \boxed{-1.929} & 0 & 0 & 0 & 0.006 & 0 \\ 0 & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & -0.002 \\ 0 & 0 & 0 & 0 & 0 & -0.951 & 0 & 0 & 0 \\ \hline 0 & -0.002 & 0 & 0 & 0 & 0 & \boxed{-535.8} & 0 & 0 \\ 0 & 0 & 0 & -0.001 & 0 & 0 & 0 & \boxed{-567} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.001 & 0 & 0 & \boxed{-570.5} \end{array}\right] \quad (27)$$

$$\tilde{B} = K_d B = \begin{bmatrix} 1.935 & 0 & 0 \\ 76.321 & 0 & 0 \\ 0 & 1.878 & 0 \\ 0 & 3.635 & 0 \\ 0 & 0 & 1.828 \\ 0 & 0 & 1.721 \\ 71.428 & 0 & 0 \\ 0 & 71.428 & 0 \\ 0 & 0 & 71.428 \end{bmatrix}, \quad \tilde{C} = C K_d^{-1} = \begin{bmatrix} 1.0000 & -0.0273 & 0 & 0 & 0 & 0 & 0.0021 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & -1.0685 & 0 & 0 \\ 0 & 0 & 1.0000 & -0.5184 & 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & -0.0509 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & -1.0627 & 0 & 0 & 0.00 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & -0.024 \end{bmatrix} \quad (28)$$

IV. DISCUSSIONS AND CONCLUSIONS

The decoupled state matrix $\tilde{A}$ in (27) is rewritten in (29) using a constant ε. Two facts need to be observed in (29). If we neglect the weak coupling factor $\varepsilon \in (-0.003, 0.031)$, then $\tilde{A}$ is essentially similar to a diagonal matrix. Moreover, the transformed system matrix $\tilde{A}$ has zero rows corresponding to the new states with $i = 1, 2, 3$, hence, RS transformation results in the decoupling and the rank reduction of the system matrix.

In part III of this work, a control algorithm would be developed to control the dynamics of the stabilizing platform.

$$\tilde{A} = \left[\begin{array}{cccccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 \\ 0 & -36.7 & 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ \hline 0 & \varepsilon & 0 & 0 & 0 & 0 & -536 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon & 0 & 0 & 0 & -570 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 & -571 \end{array}\right] \quad (29)$$

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