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Certain Multiple Series Identities

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Abstract: In this paper a theorem for general multiple series is established using Dixon's theorem and Srivastava's identities. The theorem proved in this paper provides new transformations and connections with various classes of well known hyper geometric functions and even new representations for special cases of these functions.

Keywords: Hypergeometric functions; Srivastava's triple hypergeometric functions Subject classification codes: 33C05

I. Introduction

Let (a_A) denote the sequence of A parameters given by $a_1, a_2, \dots, a_A$ in the contracted notations and $[(a_A)]_n$ denote the product of A Pochhamer symbols defined by

$$(b)_n = \frac{\Gamma(b+n)}{\Gamma(b)} = \begin{cases} 1, & \text{if } n = 0 \\ b(b+1)\dots\dots(b+n-1), & \text{if } n = 1, 2, 3, \dots \end{cases} \quad (1.1)$$

where the notation Γ denotes the Gamma function.

In 1969, Srivastava and Daoust([7, p. 454], see also [8, p. 37(21, 22)]) gave the following multivariable hypergeometric function:

$$F^{A:B^{(1)};\,\dots;B^{(n)}}_{D:E^{(1)};\,\dots;E^{(n)}}\left[\begin{matrix}[(a_A):\theta^{(1)},\dots,\theta^{(n)}]:[b^{(1)}_{B^{(1)}}:\Phi^{(1)}];\ \dots;[b^{(n)}_{B^{(n)}}:\Phi^{(n)}] \\ [(d_D):\Psi^{(1)},\dots\Psi^{(n)}]:\left[e^{(1)}_{E^{(1)}}:\delta^{(1)}\right];\dots;[e^{(n)}_{E^{(n)}}:\delta^{(n)}]\end{matrix}\quad z_1,\dots,z_n\right] \quad (1.2)$$

$$= \sum_{m_1,\dots,m_n}^{\infty} \Xi\,(m_1,\dots,m_n)\frac{{z_1}^{m_1}}{(m_1)!}\cdots\frac{{z_n}^{m_n}}{(m_n)!},$$

where for convenience,

$$\Xi(m_1,\dots,m_n) = \frac{\prod_{j=1}^{A}(a_j)_{m_1{\theta_j}^{(1)}+\dots+m_n{\theta_j}^{(n)}}\prod_{j=1}^{B^{(1)}}({b_j}^{(1)})_{m_1{\Phi_j}^{(1)}}\cdots\prod_{j=1}^{B^{(n)}}({b_j}^{(n)})_{m_n{\Phi_j}^{(n)}}}{\prod_{j=1}^{D}(d_j)_{m_1{\Psi_j}^{(1)}+\dots+m_n{\Psi_j}^{(n)}}\prod_{j=1}^{E^{(1)}}({e_j}^{(1)})_{m_1{\delta_j}^{(1)}}\cdots\prod_{j=1}^{E^{(n)}}({e_j}^{(n)})_{m_n{\delta_j}^{(n)}}}. \quad (1.3)$$

The coefficients $\theta_j^{(k)}, j = 1, \ldots, A$, $\Phi_j^{(k)}, j = 1, \ldots, B^{(k)}$, $\Psi_j^{(k)}$, $j = 1, \ldots, D$, $\delta_j^{(k)}$, $j = 1, \ldots, E^{(k)}$, for all k$\in \{1, \ldots, n\}$are zero and real constants(positive, negative)[8]

In the present paper investigation of general multiple series identities is done which extend and generalize the theorems of Bailey[1], and Pathan[2].The theorem given in Section 2 will be seen extremely useful as it provides connections with various classes of well-knownhypergeometric functions and even new representations of these functions.Some applications of this theorem are given in Section 3. Also we deduce special cases in Section 4.

and $(b_{B^{(k)}}^{(k)})$ abbreviates the array of $B^{(k)}$ parameters $b_j^{(k)}, j = 1, \ldots, B^{(k)}$; for all k$\in \{1, \ldots, n\}$ with similar interpretations for others.

II. General Multiple Series Identities

Theorem: Let $S(i, j, k, p)$be the generalized coefficient of arbitrary complex numbers, where x, y, z be complex variables and c, f be arbitrary independent complex parameters (where $2f \neq 0, \pm 2, \pm 3, \ldots$) and any values of numerator and denominator parameters and variables x, y, z leading to the results which do not make sense are tacitly excluded, then

$$\sum_{i,j,k,p=0}^{\infty} S_1(\theta_1 i + \theta_2 j + \theta_1 k + \theta_3 p) S_2(\theta_4 i + \theta_5 j + \theta_4 k) S_3(\theta_6 i + \theta_6 k + \theta_7 p)$$

$$\times S_4(\theta_8 j + \theta_9 p) S_5(\theta_{10} i + \theta_{10} k) S_6(\theta_{11} j) S_7(\theta_{12} p)$$

$$\times \frac{(-1)^k (f)_i (c)_i (f)_k (c)_k x^j y^{i+k} z^p}{i!\, j!\, k!\, p!}$$

$$= \sum_{i,j,p=0}^{\infty} S_1(2\theta_1 i + \theta_2 j + \theta_3 p) S_2(2\theta_4 i + \theta_5 j) S_3(2\theta_6 i + \theta_7 p) S_4(\theta_8 j + \theta_9 p)$$

$$\times S_5(2\theta_{10} i) S_6(\theta_{11} j) S_7(\theta_{12} p) \frac{(-1)^i (f)_i (c)_i \left(\frac{c+f}{2}\right)_i \left(\frac{1+c+f}{2}\right)_i x^j y^i z^p}{i!\, j!\, p!\, (c+f)_i} \quad (2.1)$$

$$= \sum_{i,j,p=0}^{\infty} \sum_{u=0}^{1} S_1(2\theta_1 i + 2\theta_2 j + \theta_2 u + \theta_3 p)\, S_2(2\theta_4 i + 2\theta_5 j + \theta_5 u) S_3(2\theta_6 + \theta_7 p)$$

$$\times S_4(2\theta_8 j + \theta_8 u + \theta_9 p) S_5(2\theta_{10} i) S_6(2\theta_{11} j + \theta_{11} u) S_7(2\theta_{12} p + \theta_{12} w)$$

$$\times \frac{(-1)^i x^u (f)_i (c)_i \left(\frac{c+f}{2}\right)_i \left(\frac{1+c+f}{2}\right)_i (\frac{x^2}{4})^j (\frac{y^2}{2})^i z^p}{i!\, u!\, (\frac{1+u}{2})_j (\frac{2+u}{2})_j p!\, (c+f)_i} \quad (2.2)$$

$$= \sum_{i,j,p=0}^{\infty} \sum_{u,w=0}^{1} S_1(2\theta_1 i + 2\theta_2 j + \theta_2 u + 2\theta_3 p + \theta_3 w) S_2(2\theta_4 i + 2\theta_5 j + \theta_5 u)$$

$$\times S_3(2\theta_6 i + 2\theta_7 p + \theta_7 w) S_4(2\theta_8 j + \theta_8 u + 2\theta_9 p + \theta_9 w) S_5(2\theta_{10} i) S_6(2\theta_{11} j + \theta_{11} u)$$

(2.3)

$$\times S_7(2\theta_{12}p+\theta_{12}w)\frac{(-1)^i x^u z^w (f)_i (c)_i \left(\frac{c+f}{2}\right)_i \left(\frac{1+c+f}{2}\right)_i (\frac{x^2}{4})^j (\frac{y^2}{2})^i (\frac{z^2}{4})^p}{i!\,u!\,w!\,(\frac{1+u}{2})_j(\frac{2+u}{2})_j(\frac{1+w}{2})_p(\frac{2+w}{2})_p(c+f)_i}$$

Proof. Let L denote the L.H.S of equation (2.1) .Then using the series identity [3]i.e. replacing i by $i-k$

$$\sum_{i,j,k,p=0}^{\infty} A(i,j,k,p) = \sum_{i,j,p=0}^{\infty} \sum_{k=0}^{i} A(i-k,j,k,p)$$

we may write

$$L = \sum_{i,j,p=0}^{\infty} S_1(\theta_1 i+\theta_2 j+\theta_3 p)S_2(\theta_4 i+\theta_5 j)S_3(\theta_6 i+\theta_7 p)S_4(\theta_8 j+\theta_9 p)S_5(\theta_{10} i)$$

$$\times S_6(\theta_{11}j)S_7(\theta_{12}p)\sum_{k=0}^{i}(-1)^k\frac{(f)_{i-k}(c)_{i-k}(f)_k(c)_k x^j y^i z^p}{(i-k)!\,j!\,k!\,p!}$$

$$= \sum_{i,j,p=0}^{\infty} S_1(\theta_1 i+\theta_2 j+\theta_3 p)S_2(\theta_4 i+\theta_5 j)S_3(\theta_6 i+\theta_7 p)S_4(\theta_8 j+\theta_9 p)S_5(\theta_{10} i)$$

$$\times S_6(\theta_{11}j)S_7(\theta_{12}p)\frac{(f)_i(c)_i x^j y^i z^p}{i!\,j!\,p!}\quad {}_3F_2\left(\begin{matrix}-i\ ,f,\ \ c\ ;\ 1\\ 1-f-i\,,1-c-i\end{matrix}\right)$$

Using Dixon's Theorem [4]in (2.5) we get

$$L = \sum_{i,j,p=0}^{\infty} S_1(\theta_1 i+\theta_2 j+\theta_3 p)S_2(\theta_4 i+\theta_5 j)S_3(\theta_6 i+\theta_7 p)S_4(\theta_8 j+\theta_9 p)S_5(\theta_{10} i)\times$$

$$S_6(\theta_{11}j)S_7(\theta_{12}p)\frac{(f)_i(c)_i x^j y^i z^p}{i!\,j!\,p!}\frac{\Gamma(1-{}^{i}/_{2})\Gamma(1-f-i)\Gamma(1-c-i)\Gamma(1-{}^{i}/_{2}-f-c)}{\Gamma(1-i)\Gamma(1-{}^{i}/_{2}-f)\Gamma(1-{}^{i}/_{2}-c)\Gamma(1-i-f-c)}$$

Using the identity[9]:

$$\sum_{i=0}^{\infty} A(i) = \sum_{i=0}^{\infty} A(2i) + \sum_{i=0}^{\infty} A(2i+1)$$

$$L = \sum_{i,j,p=0}^{\infty} S_1(2\theta_1 i + \theta_2 j + \theta_3 p) S_2(2\theta_4 i + \theta_5 j) S_3(2\theta_6 i + \theta_7 p)$$

$$\times S_4(\theta_8 j + \theta_9 p) S_5(2\theta_{10} i) S_6(\theta_{11} j) S_7(\theta_{12} p)$$

$$\times \frac{(f)_{2i}(c)_{2i} x^j y^{2i} z^p}{(2i)!\, j!\, p!} \frac{\Gamma(1-i)\Gamma(1-f-2i)\Gamma(1-c-2i)\Gamma(1-i-f-c)}{\Gamma(1-2i)\Gamma(1-i-f)\Gamma(1-i-c)\Gamma(1-2i-f-c)}$$

$$+ \sum_{i,j,p=0}^{\infty} S_1(2\theta_1 i + \theta_1 + \theta_2 j + \theta_3 p) S_2(2\theta_4 i + \theta_4 + \theta_5 j) S_3(2\theta_6 i + \theta_6 + \theta_7 p)$$

$$\times S_4(\theta_8 j + \theta_9 p) S_5(2\theta_{10} i + \theta_{10}) S_6(\theta_{11} j) S_7(\theta_{12} p) \frac{(f)_{2i+1}(c)_{2i+1} x^j y^{2i+1} z^p}{(2i+1)!\, j!\, p!}$$

$$\times \frac{\Gamma\left(1-i-\frac{1}{2}\right)\Gamma(1-f-2i-1)\Gamma(1-c-2i-1)\Gamma\left(1-i-\frac{1}{2}-f-c\right)}{\Gamma(1-2i-1)\Gamma\left(1-i-\frac{1}{2}-f\right)\Gamma\left(1-i-\frac{1}{2}-c\right)\Gamma(1-2i-1-f-c)}$$

$$A(2i+1) = 0 \text{as} \frac{1}{\Gamma(-2i)} = 0$$

$$L = \sum_{i,j,p=0}^{\infty} S_1(2\theta_1 i + \theta_2 j + \theta_3 p) S_2(2\theta_4 i + \theta_5 j) S_3(2\theta_6 i + \theta_7 p)$$

$$\times S_4(\theta_8 j + \theta_9 p) S_5(2\theta_{10} i) S_6(\theta_{11} j) S_7(\theta_{12} p) \frac{(f)_{2i}(c)_{2i} x^j y^{2i} z^p}{(2i)!\, j!\, p!}$$

$$\times \frac{(1-f)_{-2i}(1-c)_{-2i}(1-f-c)_{-i}}{(1-i)_{-i}(1-f)_{-i}(1-c)_{-i}(1-f-c)_{-2i}}$$

$$= \sum_{i,j,p=0}^{\infty} S_1(2\theta_1 i + \theta_2 j + \theta_3 p) S_2(2\theta_4 i + \theta_5 j) S_3(2\theta_6 i + \theta_7 p)$$

$$\times S_4(\theta_8 j + \theta_9 p) S_5(2\theta_{10} i) S_6(\theta_{11} j) S_7(\theta_{12} p) \frac{x^j y^{2i} z^p}{(2i)!\, j!\, p!}$$

$$\times \frac{(f)_i (c)_i (\frac{c+f}{2})_i (\frac{1+c+f}{2})_i (i)_i}{(c+f)_i}$$

$$= \sum_{i,j,p=0}^{\infty} S_1(2\theta_1 i + \theta_2 j + \theta_3 p) S_2(2\theta_4 i + \theta_5 j) S_3(2\theta_6 i + \theta_7 p)$$

$$\times S_4(\theta_8 j+\theta_9 p)S_5(2\theta_{10}i)S_6(\theta_{11}j)S_7(\theta_{12}p)\frac{x^j y^{2i} z^p}{j!\,p!}$$

$$\times\frac{(f)_i(c)_i\left(\frac{c+f}{2}\right)_i\left(\frac{1+c+f}{2}\right)_i(i-1)!\,i(i+1)(i+2)\dots(2i-1)}{(i-1)!\,2i(2i-1)(2i-2)\dots3\cdot2\cdot1\,(c+f)_i}$$

$$=\sum_{i,j,p=0}^{\infty}S_1(2\theta_1 i+\theta_2 j+\theta_3 p)S_2(2\theta_4 i+\theta_5 j)S_3(2\theta_6 i+\theta_7 p)$$

$$\times S_4(\theta_8 j+\theta_9 p)S_5(2\theta_{10}i)S_6(\theta_{11}j)S_7(\theta_{12}p)\frac{x^j \frac{y^{2i}}{2} z^p}{j!\,p!}$$

$$\times\frac{(f)_i(c)_i\left(\frac{c+f}{2}\right)_i\left(\frac{1+c+f}{2}\right)_i}{i!\ (c+f)_i}$$

Again applying Srivastava's identity[9],

$$\sum_{j=0}^{\infty}A(j)=\sum_{u=0}^{1}\ \sum_{j=0}^{\infty}A(2j+u)$$

in the equation (2.1)and replacing the gamma functions by Pochhamersymbols, we get

$$L=\sum_{i,j,p=0}^{\infty}\ \sum_{u=0}^{1}S_1(2\theta_1 i+2\theta_2 j+\theta_2 u+\theta_3 p)S_2(2\theta_4 i+2\theta_5 j+\theta_5 u)$$

$$\times S_3(2\theta_6 i+\theta_7 p)S_4(2\theta_8 j+\theta_8 u+\theta_9 p)S_5(2\theta_{10}i)S_6(2\theta_{11}j+\theta_{11}u)S_7(\theta_{12}p)$$

$$\times\frac{x^{2j+u}\frac{y^{2i}}{2}z^p}{(2j+u)!\,p!}\frac{(f)_i(c)_i\left(\frac{c+f}{2}\right)_i\left(\frac{1+c+f}{2}\right)_i}{i!\ (c+f)_i}$$

$$=\sum_{i,j,p=0}^{\infty}\ \sum_{u=0}^{1}S_1(2\theta_1 i+2\theta_2 j+\theta_2 u+\theta_3 p)S_2(2\theta_4 i+2\theta_5 j+\theta_5 u)$$

$$\times S_3(2\theta_6 i+\theta_7 p)S_4(2\theta_8 j+\theta_8 u+\theta_9 p)S_5(2\theta_{10}i)S_6(2\theta_{11}+\theta_{11}u)S_7(\theta_{12}p)$$

$$\times\frac{\frac{x^{2j}}{4^j}\frac{y^{2i}}{2}z^p}{}\frac{(f)_i(c)_i\left(\frac{c+f}{2}\right)_i\left(\frac{1+c+f}{2}\right)_i x^u}{i!\,u!\,(\frac{1+u}{2})_j(\frac{2+u}{2})_j p!\,(c+f)_i}$$

which is the right-hand side of (2.2).

Now applying Srivastava's identity[4]

$$\sum_{j,p=0}^{\infty} B(j,p) = \sum_{u=0}^{1} \sum_{w=0}^{1} \sum_{j,p=0}^{\infty} B(2j+u, 2p+w)$$

to(2.1) we get

$$L = \sum_{i,j,p=0}^{\infty} S_1(2\theta_1 i + 2\theta_2 j + \theta_2 u + 2\theta_3 p + \theta_3 w) S_2(2\theta_4 i + 2\theta_5 j + \theta_5 u)$$

$$\times S_3(2\theta_6 i + 2\theta_7 p + \theta_7 w) S_4(2\theta_8 j + \theta_8 u + 2\theta_9 p + \theta_9 w) S_5(2\theta_{10} i)$$

$$\times S_6(2\theta_{11} j + \theta_{11} u) S_7(2\theta_{12} p + \theta_{12} w)$$

$$\times \frac{x^u \frac{y^{2i}}{2} z^w}{u!\, w!} \frac{(-1)^i (f)_i (c)_i \left(\frac{c+f}{2}\right)_i \left(\frac{1+c+f}{2}\right)_i}{i!\ (c+f)_i (\frac{1+u}{2})_j (\frac{2+u}{2})_j (\frac{1+w}{2})_p (\frac{2+w}{2})_p}$$

which is the right-hand side of (2.3)

III. Applications of theorems 2.1 – 2.3

3.1. In theorem 2.1 and 2.2 setting$\theta_1 = \theta_2 = \theta_3 = \cdots = \theta_{12} = 1$ and

$$S_1(i+j+k+p) = S_3(i+k+p) = S_4(j+p) = S_7(p) = 1,$$

$$S_2(j+i+k) = \frac{[(a_A)]_{j+i+k}}{[(b_B)]_{j+i+k}}, S_5(i+k) = \frac{[(d_D)]_{i+k}}{[(e_E)]_{i+k}},\quad S_6(j) = \frac{[(g_G)]_j}{[(h_H)]_j}, \text{and } z = 0,$$

we get

$$F^{(3)} \left[\begin{matrix} (a_A) :: -;\ (d_D);\ -: (g_G);\ c;\ f;\ c;\ f; \\ (b_B) :: -;\ (e_E);\ -: (h_H); \end{matrix} \quad x, y, -y \right]$$

$$= X_{B:H;\ 2E+1}^{A:G;\ 2D+4} \left[\begin{matrix} (a_A): (g_G);\ \Delta(2;\ c+f), \Delta[2;\ (d_D)];\ c;\ f; \\ (b_B): (h_H);\ c+f, \Delta[2;\ (e_E)]; \end{matrix} \quad x, \frac{y^2}{4^{E-D+1}} \right] \quad (3.1)$$

$$= \sum_{u=0}^{1} \frac{[(a_A)]_u [(g_G)]_u x^u}{[(b_{\boldsymbol{B}})]_u [(h_H)]_u u!} F_{2B:2H+1;\ 2E+1;}^{2A:2G;\ 2D+4} \times$$

$$\left[\begin{matrix} [\Delta(2;\ (a_A)+u]: c;\ f;\ \Delta[2;\ (g_G)+u], \Delta(2;\ c+f), \Delta[2;\ (d_D)]; \\ \Delta[2;\ (b_B)+u]: \Delta^*[2;\ 1+u], \Delta[2;\ (h_H)+u];\ c+f, \Delta[2;\ (e_E)+u]; \end{matrix} \quad \frac{4^{(A+G)} x^2}{4^{B+H+1}}, \frac{4^{(A+D)} y^2}{2^{B+E+1/2}} \right]$$

(3.2)

Provided the denominator parameters are neither zero nor negative integers and for convenience, the symbol $\Delta(m; b)$ abbreviates the array of m parameters given by

$$\frac{b}{m}, \frac{b+1}{m}, \frac{b+2}{m}, \ldots, \frac{b+m-1}{m}, \text{ where}= 1,2,3, \ldots .$$

The asterisk in$\Delta^*(N;\ j+1)$represents the fact that the (denominator) parameter N/N is always omitted for $0 \le j \le (N-1)$so the set$\Delta^*(N;\ j+1)$contains only $N-1$ parameters[9].

3.2. *In Theorem 2.3, setting* $\boldsymbol{\theta_1 = \theta_2 = \theta_3 = \cdots = \theta_{12} = 1}$*and*

$$S_2(i+j+k) = S_3(i+k+p) = 1, S_5(i+k) = \frac{[(d_D)]_{i+k}}{[(e_E)]_{i+k}},$$

$$S_1(i+j+k+p) = \frac{[(a_A)]_{i+j+k+p}}{[(b_B)]_{i+j+k+p}},$$

$$S_4(j+p) = \frac{[(m_M)]_{j+p}}{[(n_N)]_{j+p}}, \quad S_6(j) = \frac{[(g_G)]_j}{[(h_H)]_j}, S_7(p) = \frac{[(q_Q)]_p}{[(r_R)]_p} \textit{and } z\textit{=0,}$$

we get

$$F^{(4)}\left[\begin{matrix}(a_A) :: c;\ (d_D);\ (g_G);\ (m_M): f;\ (d_D);\ (q_Q);\ (m_M);\ c;\ f; & y, x, -y, z\\ (b_B);\ (e_E);\ (h_H);\ (n_N);\ (e_E);\ (r_R);\ (n_N); & \end{matrix}\right]$$

$$= \sum_{u=0}^{1}\sum_{w=0}^{1} \frac{[(a_A)]_{u+w}[(m_M)]_{u+w}[(g_G)]_u[(q_Q)]_w x^u z^w}{[(n_N)]_{u+w}[(b_B)]_{u+w}[(h_H)]_u[(r_R)]_w u!\, w!} \times$$

$$F^{(3)}\left[\begin{matrix}\Delta[2;\ (a_A)+u+w] :: -;\ \Delta[2;\ (m_M)+u+w];\ -: \Delta(2;\ c+f), \Delta[2(d_D)];\ c;\ f;\ \Delta[2;\ (g_G)+u];\\ \Delta[2;\ (b_B)+u+w] :: -;\ \Delta[2;\ (n_N)+u+w];\ -: c+f, \Delta[2;\ (e_E)];\ \Delta^*(2;\ 1+u),\end{matrix}\right.$$

$$\left.\begin{matrix}\Delta[2;\ (q_Q)+w]; & \\ \Delta[2;\ (h_H)+u];\ \Delta^*(2;\ 1+w)\Delta[2;\ (r_R)+w]; & \frac{4^{A+D}y^2}{4^{1/2+B+E}}, \frac{4^{A+M+H}x^2}{4^{1+B+N+H}}, \frac{4^{A+M+Q}z^2}{4^{1+B+N+R}}\end{matrix}\right]$$

(3.3)

IV. Special Cases:

i. In (2.1) setting

$$S_1(\theta_1 i + \theta_2 j + \theta_1 k + \theta_3 p) = S_3(\theta_6 i + \theta_6 k + \theta_7 p) = S_4(\theta_8 j + \theta_9 p)$$
$$= S_5(\theta_{10} i + \theta_{10} k) = S_7(\theta_{12} p) = 1$$

$$\sum_{i,j,k,p=0}^{\infty} S_2(\theta_4 i+\theta_5 j+\theta_4 k)\times S_6(\theta_{11}j)\frac{(-1)^k(f)_i(c)_i(f)_k(c)_k x^j y^{i+k} z^p}{i!\,j!\,k!\,p!}$$

$$=\sum_{i,j,k,p=0}^{\infty} S_2(2\theta_4 i+\theta_5 j)S_6(\theta_{11}j)\frac{(-1)^i(f)_i(c)_i\left(\frac{c+f}{2}\right)_i\left(\frac{1+c+f}{2}\right)_i x^j y^i z^p}{i!\,j!\,p!\,(c+f)_i} \quad (4.1)$$

ii. In equation (4.1) , $\theta_4=\theta_{11}=1,\theta_5=2,\ S_2(j+i+2k)=(a)_{i+j+2k}, S_6(j)=\frac{1}{(b)_j}$, we get

$$\sum_{i,j,k,p=0}^{\infty} (a)_{i+j+2k}\frac{1}{(b)_j}\ \frac{(-1)^k(f)_i(c)_i(f)_k(c)_k x^j y^{i+k} z^p}{i!\,j!\,k!\,p!}$$

$$=\sum_{i,j,p=0}^{\infty} (a)_{2j+2i}\frac{1}{(b)_j}\frac{(-1)^i(f)_i(c)_i\left(\frac{c+f}{2}\right)_i\left(\frac{1+c+f}{2}\right)_i x^j y^i z^p}{i!\,j!\,p!\,(c+f)_i} \quad (4.2)$$

iii. In (2.1) setting

$$S_2(\theta_4 i+\theta_5 j+\theta_4 k+\theta_3 p)=S_3(\theta_6 i+\theta_6 k+\theta_7 p)=S_4(\theta_8 j+\theta_9 p)$$

$$=S_5(\theta_{10}i+\theta_{10}k)=1,$$

$$S_1(i+j+k+p)=\frac{[(a_A)]_{i+j+k+p}}{[(b_B)]_{i+j+k+p}}, S_6(j)=\frac{[(g_G)]_j}{[(h_H)]_j}, S_7(p)=\frac{[(d_D)]_p}{[(e_E)]_p}$$

we get

$$\sum_{i,j,k,p=0}^{\infty}\frac{[(a_A)]_{i+j+k+p}}{[(b_B)]_{i+j+k+p}}\frac{[(g_G)]_j}{[(h_H)]_j}\frac{[(d_D)]_p}{[(e_E)]_p}\frac{(f)_i(c)_i(f)_k(c)_k(-1)^i x^j y^{i+k} z^p}{i!\,j!\,k!\,p!}$$

$$=\sum_{i,j,k,p=0}^{\infty}\frac{[(a_A)]_{2i+j+p}}{[(b_B)]_{2i+j+p}}\frac{[(g_G)]_j}{[(h_H)]_j}\frac{[(d_D)]_p}{[(e_E)]_p}\ \frac{(-1)^i(f)_i(c)_i(\frac{c+f}{2})_i(\frac{1+c+f}{2})_i x^j y^i z^p}{i!\,j!\,p!\,(c+f)_i} \quad (4.3)$$

V. References

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